The relativity of space and time

21.1 Absolute or relative time.

The passage time is something we are all aware of and for some of us there are times when there never seems to be enough. However, when we try to define what it is we are measuring it becomes obvious that time is rather mysterious. Today clocks tell us how many seconds something takes with ever increasing accuracy, but this does not tell us what it is that the clock is actually measuring. Time has always been thought about by philosophers, writers, poets and there is a great deal of literature on the topic.

Imagine yourself back in the time before clocks were invented. Actually this is not all that long ago, except for the use of the rotation of the Earth and sundials in some cultures. It was only four hundred years ago that Galileo was using his pulse to time a pendulum. He assumed that his pulse rate did not change during an experiment and did not question the nature of what he was actually measuring between heartbeats. Actually to most of us days vary in length, seasons come and go and time is more a more personal or subjective 'thing'. If you are busy, time seems to pass quickly and if you are inactive and bored, time seems to pass very slowly. When we remember the past, the busy times seem to have lasted longer than the boring times and many older people will tell you that time passes more quickly as you get old. Time is subjective and may even be a construct of consciousness as the quote below hints.

"For when I was a babe and slept, time crept,
When I was a boy and laughed and talked, time walked.
Then when the years saw me a man, time ran.
But as I older grew, time flew.
Soon I shall be passing on, then, time will be gone
O Jesus, when death comes,
Nothing will matter - but You."

Guy Dentreath

Sometimes we experience a sort of timelessness, when time seems to stand still. At other times events may seem to take place in slow motion. Stranger still we sometimes experience **deja vu.** Certainly time does **not** seem to pass at a uniform rate and our perception of time is different to that measured by clocks. Some cultures, such as the Mayan in South America, believed that time did indeed pass at varying rates and even that time is cyclic and repeats itself, which is an idea developed in Europe as well.

Stand still, you ever moving spheres of heaven,
That time may cease, and midnight never come;
Fair Natyure's eye, rise, rise again, and make
Perpetual day; or let this hour be but
A year, a month, a week, a natural day,
That Faustus may repent and save his soul!
O lente, lente currite, noctis equi!

Another Orpheus sings again, and loves,
and weeps, and dies;
A new Ulysses leaves once more
Calypso for his native shore

The world's great age begins anew,
The golden years return.
The Earth doth like a snake renew
her winter weeds out worn.
A loftier Argo cleaves the main,
Fraught with a later prize;

Another Athens shall arise, and to remoter time
Bequeath, like sunset to the skies
the splendour of it's prime.

P.B. Shelley.

"And the end and the beginning were always there
Before the beginning and after the end"

T.S.Elliot.

It was the invention of clocks that led to todays belief that time passes at a uniform or **absolute** rate. Personal time seems to be relative to the things happening. Actually all clocks are simply a series of repeated events, such as a pendulum swinging to and fro, the maximum height of the sun at noon, an oscillation of a wave, a hand passing the number 12 on a watch. We **assume** that the time interval between these events is constant but, if there was only one clock, how would we ever know if it slowed down ? Today even the most accurate clocks cannot be checked against any **absolute** standard for the passage of time. Consider the situation of there being just two fundamentally different types of clock, such as an atomic clock and a pendulum operated clock. If they disagree, how do we know which one is wrong or right ? We simply do not know and can never know. We can agree to use a particular clock as a standard, but this does not tell us anything about the nature of time itself. Having agreed on a particular clock, we must then agree that **time is what the clock measures**, and that is all we know about time. Today this clock is the Caesium atomic clock. It was Isaac Newton who first put forward a formal statement about the idea of absolute space and time and his view has held sway with the general population until today, where most people still believe it to be not only true but self evident

> ***"Absolute, true and mathematically, time of itself and from its nature flows equably without relation to anything external. All motions may be changed but the flow of time is not liable to change."***
>
> **I.Newton**

This idea did not persuade all philosophers and the idea that time was relative was put forward long before Einstein. In Europe at the same time that Newton was publishing his great work Principia, Leibnitz had the insight to see that space and time were relative and that clocks do not actually measure anything.

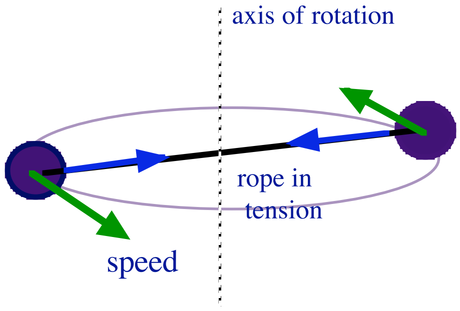
> ***"I hold space to be merely relative, as time is. I hold it to be an order of co- existences, as time is an order of successions. Instants, considered without the things* [events], *are nothing at all, they consist only in the successive order of the things."***
>
> **Leibnitz**

21.2 Absolute or relative space.

Space is also something that we take for granted. How do we define space ? In one dimension we might say that length is measured by a ruler and that it is absolute, in the sense that it is unchanging and the same for all observers regardless of their motion. This is true to a very good approximation in everyday experiences on Earth, where we do not come across very high speeds or enormous gravitational fields. However, we must see that it is an **assumption** and not necessarily a fact. It is often not realised that, if space were stretched or compressed, then all rulers would alter by the same amount. Therefore a person in that space would not register the change. Today when scientists deal with high speeds it is **not true** and we will see in the next section that distances in space depend on who is measuring them. We would also see that objects moving w.r.t. an observer are squashed in the direction of the movement.

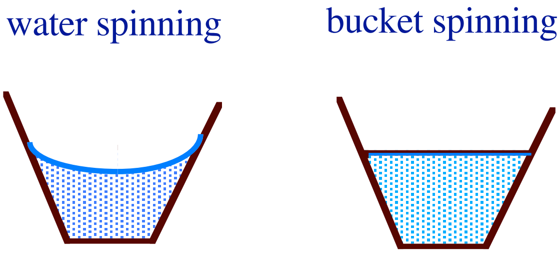
The notion of absolute space and time, that Isaac Newton required in order to explain inertia and deduce his laws of motion, lasted for hundreds of years, although not without strong argument from the relativists such as Leibnitz and E. Mach. In fact he had realised that uniform motion through his absolute space was not detectable. This is in itself a **contradiction**, as is his statement on time quoted above. It means that no observer of events involving other objects moving in his frame can say that [he] is stationary and they are moving. That is the same as saying that the only motion detectable is relative motion. However, Newton persisted with the concept of an absolute reference frame for space and a universal time. He presented rotational motion and centrifugal forces as evidence for an absolute reference frame. The great man was aware of the difficulties associated with such assumptions but he had to have working definitions. Having developed the idea of inertia, Newton had to explain how an object could resist changes in motion. He did this by postulating that space [and time] were **absolute** and that inertia did not depend on any interaction, but was an **intrinsic property of matter**. Uniform motion in this space was undetectable but **changes in the motion** relative to this absolute space were detectable and were manifested by the need for a force. He suggested that purely relative motion could not give rise to forces and he used two simple experiments to show this. Einstein was to call this type of thinking a 'gedanken [thought] experiment'. Consider the following two situations.

Fig.21[i] Two masses tied together by rope.



Imagine that the masses are spinning in a huge void. The rope will be still taut but what are the masses spinning relative to, if not absolute space ? Why is rotational motion detectable and linear motion completely undetectable. ?

Fig.21[ii] A spinning bucket of water.



If the water spins in the bucket the surface is curved. If the bucket spins around the water the surface is flat.

Therefore when the surface of the water curves in the first case it must spin relative to some absolute reference frame.

Relative space.

On the continent of Europe physics had taken a different path. **Ernst Mach** held the view that the only **real** [possible] measurements were **relative** and that to talk of an absolute reference frame was wrong because it **could not be tested.** His view was that inertia was due to the **interaction** with the rest of the universe and it was a relativistic phenomenon. The universe was not very big in those days.

The relativists, such as Leibnitz, argued that it was pointless defining something that could not be detected. If time is “without relation to anything external” then how can we ever know if it changes ? Space is also relative not absolute.

21.3 Relativity confirmed by experimental evidence.

When J.C. Maxwell derived the equations of EM waves this led him to postulate the existence of an **aether** [ether], to enable the propagation of the waves. A perfect vacuum could not have any properties whatsoever. The ether was a strange medium, supposedly having zero density and viscosity and only detectable by observing the light waves which it carried. The existence of the ether would of course define an **absolute reference frame** for space itself.

The speed of light through this medium should be constant relative to the ether, like sound waves have a constant speed through the air. Observers who are moving through the ether should find different values for the speed of light. Inevitably this led to a search for this strange medium by trying to measure the motion of the Earth through space. The rest is history, no experiment could detect any motion whatsoever. The speed of light and all EM waves in a vacuum is the same regardless of the motion of the source or the observer. The consequences of this were mind bending.

If absolute space did not exist and all measurements were only relative then we shall see that the passage of **time must also be relative**. We mean by this that time passes slower for observers moving relative to us. Initially various attempts were made to explain the experimental evidence that the speed of light was constant [to all observers] by involving contractions of length in the direction of the motion. This was because the notion that the passage of time was relative [and not absolute] could not be accepted. These length contractions, which were first put forward by **H.Lorentz**, had no experimental or theoretical basis and were simply a way of getting the answer that was wanted. Not the best scientific method**!** The concept of relative time was too radical and indeed even today, nearly a century after Einstein's famous paper on special relativity was published, the vast majority of people still think of time as ticking away at a constant and unalterable rate.

Albert Einstein’s paper on the special theory of relativity deals with the consequences of this evidence about the speed of light for observers moving with a constant relative velocity. He had come to the same conclusion about the speed of light from philosophical considerations and did not know of the result of the Michelson and Morley experiment. Einstein had thought about the changes he should see if he could travel at the speed of light alongside a light beam. At this time there was not a theoretical limit to speed. He realised that he would see the electric and magnetic fields at any point as constant, but varying with distance along the beam. This is of course impossible as there are no charges present. He concluded that it was impossible to travel at the speed of light and then went further to say that the observed speed of light could not be altered in any way. He argued that if one can never reach the speed of light **then one cannot even begin to approach it.**

We will see that space and time become very closely linked and time is really a fourth dimension, in no way different to the accepted three dimensions of Euclidian space. In this universe, time for one observer can be a distance for another. Events happen, but in a way that is really outside of what we call time.

> ***"It is more natural to think of reality as a four dimensional existence, instead of, as hitherto, the evolution*** **[in time]** ***of a three dimensional existence."***
>
> **A. Einstein.**

21.4 Some commonly asked questions.

[i] Is time only a macroscopic phenomenon ?

At the sub-atomic level it is difficult to think of time existing for, say, a proton. Events for a proton are interactions with other sub-atomic particles. Imagine, if you can, two such events. A collision with an alpha particle followed by a collision with a neutron. The proton has no other interactions in between, no other input of any kind. It cannot 'see' since that requires incoming light photons. It therefore has no perception of space. As for time it only 'knows' that two events occurred in a particular order. The notion of a measured interval of 'time' is impossible for the proton. Indeed special relativity shows us that these two events can occur simultaneously for one external observer or widely separated in time by another. The world of the sub atomic particle is really one that is somehow outside of time and space.

> ***" Every place is interconnected with all other places and all other times by these virtual processes, whose end result is somehow built into the initial event. Events 'happen' in some sort of totality, essentially independent of flow, of sequence. The easiest way to feel this is to think of such events as simply 'being'. "***
>
> **M.Shallis**

[ii] Is memory the key ?

If we had no memory of the past, then surely we could not know of the passage of time. There are a few unfortunate people with some brain damage that have lost all memory of their past. They live in a perpetual 'now' and are not aware of the passage of time.

> ***"It is a four dimensional continuum which is neither 'space' nor 'time'. Only the consciousness that passes on in one portion of this world experiences the detached piece which comes to meet it and passes behind as history, that is, as a process going forward in time and taking place in space."***
>
> **H.Weyl.**

[iii] What is meant by 'now' ?

The concept of now must surely be reliant on consciousness. Philosophers and physicists do not attribute any significance to the concept of now. We will see that there is no such thing as simultaneous events at places separated in space.

> ***"That it is a death, that it is the death of Anne Stuart, that it has such causes and effects never changes. At the last moment of time, if time has a last moment, it will still be the death of a queen. And in every respect but one, it is equally devoid of change. It was once an event in the future, became an event in the near future, at last it was present, then it became past and will always remain past."***
>
> **J.M.E.McTaggart.**
> [Philosopher; Cambridge]

[iv] Is time a construct of the mind ?

If time is not evident at the atomic level, then the laws of thermodynamics must be the key ? Entropy and the trend to disorder is a process that we 'see' as time. We know that the role of the observer is paramount and it would be not at all unreasonable to answer yes to the question.

[v] Can time travel backwards ?

At the macroscopic level we must say, **no**. Events do usually have a cause and effect cannot precede cause. Also we see an inevitable trend towards disorder or randomness. This is the second law of thermodynamics at work. We do not see a broken dinner plate rise from the floor and reassemble. A discussion of this topic could take a complete book and we will not develop this question further. However, at the sub atomic level of fundamental particles it seems that the direction of time is not obvious. Events happen without a cause and can be described equally well if the direction of time is reversed.

[vi] Is time travel possible ?

The answer to this question is **yes**, but only forward in time. Special Relativity shows us that time passes slower for the moving observer. Therefore if you were to go on a high speed and very long journey and then come back to Earth you will find that, what for you was say one month, was many years on the Earth. You will have moved into the future.

[vii] Is time the fourth dimension ?

Mathematically, in the laws of physics [the universe], time has become like the dimensions of space. We shall see that time can stand still, that what one observer sees as a time interval, another 'sees' as a separation in space. At the speed of light time ceases to exist.

21.5 Einstein's special theory of relativity.

What follows is not very difficult mathematics but it does require very careful thought. Einstein's general theory of relativity deals with accelerating frames of reference and this will not be dealt with to any depth in this book. There is a short section at the end of the chapter. The electromagnetic effects of moving charges and electric currents are covered in the next chapter. A. Einstein's great insight was to discard absolute time [and space] and then look for the consequences. The **two postulates** that he made were :-

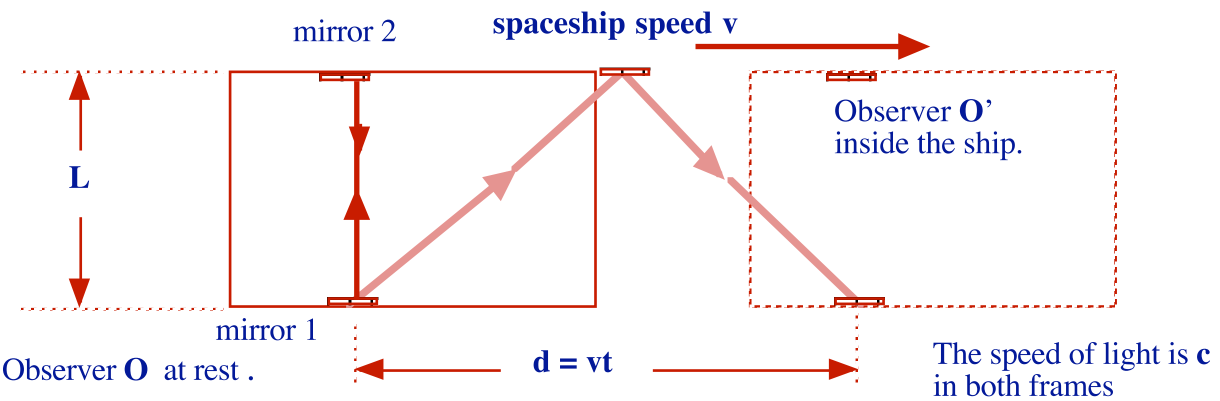
[i] The laws of physics are the same in all inertial [non accelerating] frames of reference. This means you cannot detect absolute motion or it is not possible to say that you are stationary an another object is moving. The reverse is equally true. This does not mean that all observers will agree on the actual values of quantities such as mass, length, time [interval], force, etc. It means that the **laws** of physics connecting these quantities are the same.

[ii] The speed of light is the same to all observers in all inertial frames. This follows from the first postulate if we include EM waves in the laws of physics, which we must.

21.6 Time dilation.

These assumptions lead to clocks and therefore time itself running slower in any frame of reference moving relative to the observer. This is quite easily derived. What is difficult, is to accept that the speed of light is actually the same to all observers, with all of the consequences. However, it is not just a theory, it is an **experimental fact** and we **must** accept it. It leads to a complete revision of the laws of physics, and the changes have all had experimental confirmation. Time dilation was the first consequence of Einstein's theory that was tested by experiment and the changes were in complete agreement with the theory.

Fig.21[iii]: Time dilation



Imagine a spaceship passing overhead. Inside is an observer **O'**, who measures the time **t'** for a beam of light to pass from mirror 1 to mirror 2 and back again. **O'** will see the light go straight up and down. The time for the event measured by **O'** is of course simply $t' = {}^{2L}/_{c}$

Now consider the same event seen by a ground based observer **O**. The light will follow the path shown in the diagram for [him] and it travels a greater distance. Observer **O** will therefore calculate the duration or time for the event [the passage of the light] as

$$t = \frac{2\left[L^2 + \left({}^{vt}/_{2}\right)^2\right]^{\frac{1}{2}}}{c}$$

If we squared both sides of the two expressions for the time and eliminate **L** we get :-

$$t = \frac{t'}{\sqrt{1 - v^2/c^2}}$$

equation **[1]**

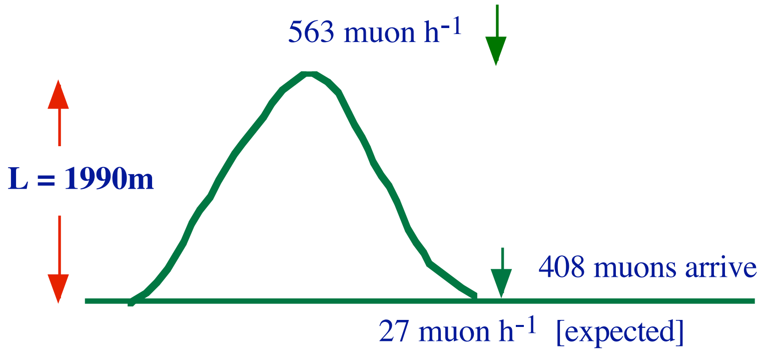
Where **t'** is the time observed in the moving frame by **O'** and **t** is the time measured by the stationary **O.** Thus $t' < t$ which means time passes slower in the spaceship.

n.b. The effect is mutual. If, at the same time, **O'** observes **O** [he] will see that time passes slower for **O**. **i.e.** $t < t'$

The Frisch - Smith Experiment.

Time dilation can be measured for the sub atomic particles called **muons**, which are rather like massive electrons. They are created in the atmosphere by interaction with high energy cosmic rays. Muons have a very short half-life of **1.53×10^{-6}s** when they are at rest with respect to the observer, however, if the muons are observed when travelling at near the speed of light this will be much longer for the stationary observer.

Fig.21[iv] Muon decay.



The time to reach sea level is 6.7×10^{-6}s in the frame of the Earth; about 4.4 half-lives.

The time in the frame of the muons is only 7.5×10^{-7}s, some 0.5 of a half life.

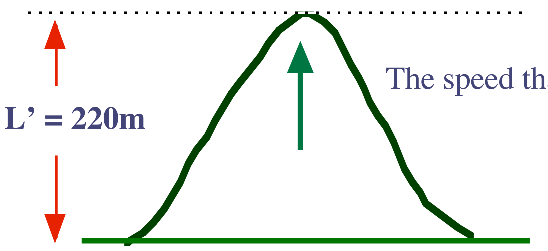
In a famous experiment, performed in the sixties, D. Frisch and J. Smith counted the number of muons at the top of a mountain that had a speed of **0.99c** vertically downwards and then the number of these surviving at sea level. The expected number at sea level a fraction of a second later would be only about 5% of those starting at the top of the mountain if time was absolute. They found that over 70% reached sea level, which was very close to the number predicted by special relativity, if in fact time did pass more slowly for the muons.

21.7 Length contraction.

[i] Longitudinal contraction.

Let us look at the experiment above in the frame of reference of the muons, by perhaps imagining an observer **O'** in a spaceship travelling at 0.99c beside them. The speed that the mountain passes by the now stationary space ship is 0.99c. It should be obvious that the speed will be the same by symmetry.

Fig.21[v]. The frame of reference of O'.



563 muon h^{-1} existing

The speed that the mountain passes is 0.99**c**

The time **t'** from top to bottom in the frame of O' is only 7.5×10^{-7}s

408 muons h^{-1} remaining

We find that the distance travelled, as measured in this frame, is not 1990m but only about 220m. This can be seen if we calculate the distance from [distance = speed x time]:-

$$L' = v\,t' = [0.99 \times 3.00 \times 10^{8}] \times [7.5 \times 10^{-6}] = 220\text{m}$$

Therefore it seems that what an Earthly observer sees as time dilation, an observer travelling with the muons sees as a length contraction. He sees the distance down to the sea **L'** as much less than the observer on the mountain or standing at sea level. The relationship showing the amount of length contraction is therefore.

$$L = v\,t \quad \text{and} \qquad L' = v\,t' = v\,t\sqrt{1 - v^2/c^2}$$

$$L' = L\sqrt{1 - v^2/c^2}$$

equation **[2]**

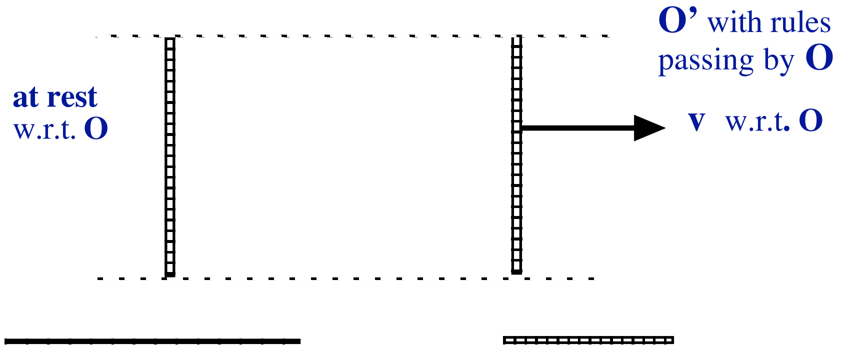
A different observer moving relative to both would see a different time **and** length.

Thus space and time are relative and **linked**. We refer today to **spacetime as an entity** and not two separate phenomena. It is important to realise that this effect is **also mutual.** The ground based observer sees the spaceship squashed and much shorter than the pilot does. The pilot uses a length contracted ruler to measure the length of his ship and does not observe any length contraction of the ship.

[ii] Transverse lengths.

There is no length contraction perpendicular to the motion of the observer. If there was a contraction in this direction then it would be possible to tell which of two observers was moving fastest relative to absolute space, which is not possible. Imagine two observers initially at rest w.r.t each other and holding a metre rule up vertically. If they then pass each other in the horizontal direction later and one rule was shorter than the other it would be obvious who moved the faster. **Faster** implies w.r.t something else !!

Fig.21[vi]. Transverse and longitudinal lengths.



Longitudinally **O** sees **O'**s rule contracted; and remember, the effect is mutual, O' sees O contracted

21.8 The Lorentz transformations.

If we wish to change our frame of reference to one that is moving relative to us at speed **v** the equations, which were first put forward by **H.A.Lorentz** are derived :-

[i] Transformation equation for length.

Consider a beam of light emitted at the origin **O**. This moves in the positive **x** direction and hits an object at **P.** **O'** is moving at a speed **v** relative to **O** and the event E_0 also occurs at the origin for **O'.** The times taken to reach **P** are **t** and **t'**

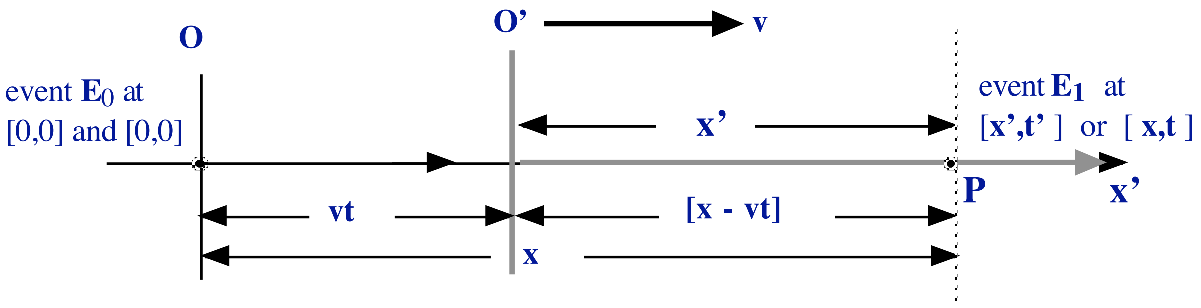
Point **P** is at **x** for observer **O** and **x'** for observer **O'**

O sees **O'P** as **[x - vt]** but observer **O'** measures length with a contracted ruler and

therefore the length that **O'** measures is greater than that seen by **O.** $x' > [x - vt]$

Fig.21[vii] The frames of O and O'

E_1 occurs when $t = t' = 0$ and $x = x' = 0$



therefore
$$x' = \frac{x - vt}{[1 - v^2/c^2]^{1/2}}$$

$y' = y$ and $z' = z$

[ii] Transformation of time

The times that the event E_1 occurs are simply $t' = {}^{x'}/_{c}$ and $t = {}^{x}/_{c}$

Using equation [1] and dividing top and bottom by **c** we get :-

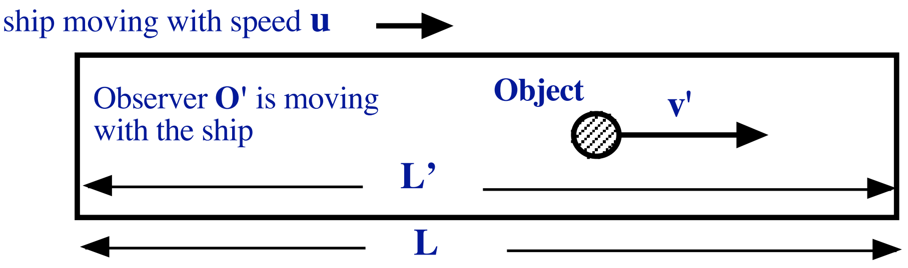
$$t' = \frac{[x - vt]}{c\,[1 - v^2/c^2]^{1/2}} = \frac{[t - vt/c]}{[1 - v^2/c^2]^{1/2}} = \frac{[t - vx/c^2]}{[1 - v^2/c^2]^{1/2}}$$

$$t' = \frac{[t - vx/c^2]}{[1 - v^2/c^2]^{1/2}}$$

[iii] Relativistic addition of velocities.

Consider a spaceship moving with a speed **u** relative to and observer **O** on the ground. The velocity of an object relative to an observer **O'**, who is inside the space ship, is **v'**. When it is observed by the ground based observer **O** the speed of the object observed is **not [u + v']**.

Fig.21[viii]. Relativistic addition of velocities.



Observer **O** $v = L/t$ and for observer **O'** $v' = L'/t'$

Observer **O** is stationary with respect to the ground and observes the speed of the object to be **v** as given by :-

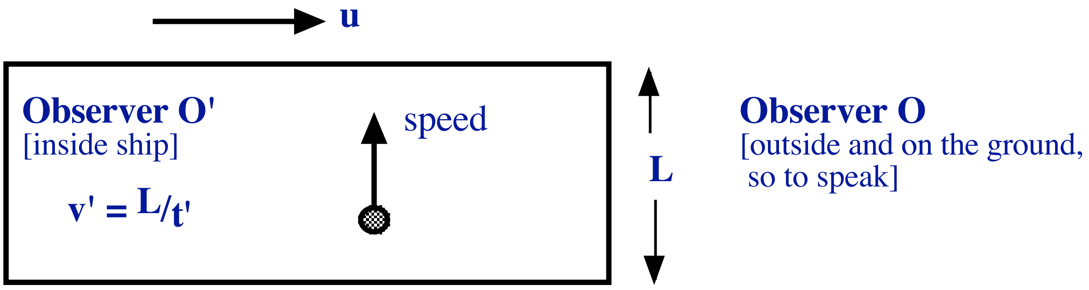
$$v = \frac{u + v'}{[1 + v'u/c^2\ ]} \qquad \text{equation [3]}$$

This is not at all obvious. Everyday experience tells us that we do just add the velocities, and for low speeds this is a very close approximation and the difference is too small to be measured. In order to derive the expression for speeds in different frames of reference we must use the Lorentz transformations for length and time. This is not a short derivation, although the actual maths is easy. The derivation of equation [3] will not be covered here.

[iv] Transverse transformation of velocity.

What is of more interest is the effect of relative motion on a speed **perpendicular** to the motion of the observer.

Fig.21[ix]. Transformation of velocity perpendicular to the relative motion.



Since there is no length contraction, except in the direction of the relative motion, then the only effect of the motion is on the time measured by the clock in the frame of the observer **O'**. This will run slow compared to that of **O** on the ground and hence the time that **O'** measures for the object to travel say the distance **L** is less and the speed measured by **O'** will be greater than that measured by **O**.

$$\textbf{but} \quad t' = t\sqrt{1 - u^2/c^2}$$

$$\text{Therefore} \quad v = v'\sqrt{1 - u^2/c^2}$$

equation **[4]**

This expression will be used to derive the expression for relativistic mass in the next section.

21.9 Relativistic momentum.

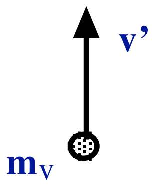
It is important that the laws of physics are the same in all frames of reference or the universe would not make sense. These transformation equations are constructed on the assumption that the laws are universal. Thus the law of **conservation of momentum is true in all frames**, although the actual values of the momenta are different. The conservation of momentum is the ***a priori* assumption of Newton**, and this assumption also defines inertial mass [as the quantity '**m**' in **p = mv**]. Although we nearly always 'weigh' objects to find their mass, fundamentally mass should be measured by its inertia in a collision experiment. The definition of force, as the rate of change of momentum, follows and the laws of motion all derive from this. We will see that if momentum is to be conserved in all frames of reference, then the inertial mass must vary with speed. Momentum in any frame is **p = mv** but where **m** is a function of **v ;** and **v** varies for different observers. We will see in the next section that **mass and energy** are also intimately connected.

21.10 The transformation of transverse momentum.

The magnitudes of most quantities change when measured from the point of view of different frames of reference. However in the case of the momentum perpendicular to the relative velocity of two observers there is no change.

This is of particular interest in the next section on electromagnetism and the proof is as follows:

Fig.21[xvi] Observer O' stationary [in the horizontal direction].

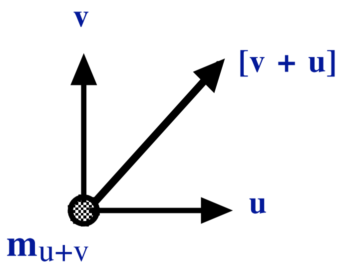


Momentum $p' = m_{v'}\, v'$

using the expression for relativistic momentum and equation [5] we have

$$p' = m_{v'}\, v' = \frac{m_o\, v'}{\sqrt{1 - \frac{v'^2}{c^2}}}$$

Fig.21[xvii] Observer O moving [horizontally] **left at speed, u.**



Vertical momentum $p = m_{[v+u]}\, v$

$$p = m_{[v+u]} \cdot v = \frac{m_o\, v}{\sqrt{1 - \frac{u^2 + v^2}{c^2}}}$$

We can show that in the vertical direction $p = p'$ by using equation [4] :-

since $v = v'\sqrt{1 - u^2/c^2}$ then

$$p' = \frac{m_0 v}{\left[1 - \dfrac{v^2}{c^2\left[1 - u^2/c^2\right]}\right]^{1/2}\left[1 - u^2/c^2\right]^{1/2}}$$

$$\left[\frac{p'}{p}\right]^2 = \frac{\left[1 - \dfrac{u^2 + v^2}{c^2}\right]}{\left[1 - \dfrac{v^2}{c^2\left[1 - u^2/c^2\right]}\right]\left[1 - u^2/c^2\right]}$$

multiply top and bottom by c^2 and we get

$$\left[p/p'\right]^2 = \frac{\left[c^2 - u^2 - v^2\right]}{\left[1 - \dfrac{v^2}{\left[c^2 - u^2\right]}\right]\left[c^2 - u^2\right]} = 1$$

Thus $p/p' = 1$ and hence $p = p'$

Hence transverse momentum remains the same under a Lorentz transformation

21.11 The transformation of force.

A force is necessary to cause a change in the motion of a mass. Force is defined as equal to the rate of change of momentum of a mass.

$F = dp/dt = d/dt\,[mv]$ where **m** and **v** are measured in any frame.

Momentum is **conserved** in **all frames** and therefore action and reaction will always be equal and opposite. The laws of physics are universal laws.

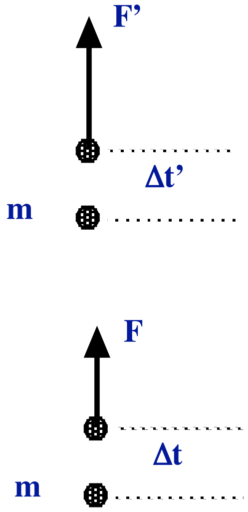
$F = dp/dt$ hence $\Delta p = F\,\Delta t$

21.12 The transformation of transverse forces.

If a force acts on the mass, which is initially at rest in the frame of **O'**, for a short time and causes a change of momentum, then using the fact that momentum is unchanged when measured by different observers, it follows that for an observer **O** moving relative to the mass the force **F** will be less, since time passes slower for the moving observer.

If a force acts on the mass, which is initially at rest in the frame of **O'**, for a short time and causes a change of momentum, then using the fact that transverse momentum is unchanged when measured by different observers, it follows that for an observer **O** moving relative to the mass the force **F** will be less, since time passes slower for the moving observer.

Fig.21[xviii] Forces measured in different frames [perpendicular to the relative motion].



Observer **O'** stationary w.r.t. **m.**

$\Delta p' = F'\,\Delta t'$ **n.b.** $\Delta t' < \Delta t$

As seen by an observer **O** moving left at speed **u.**

$\Delta p = F\,\Delta t$

$\Delta p = F\,\Delta t = \Delta p' = F'\,\Delta t'$

The force is larger in the frame of reference of **O'**, where the mass is at rest relative to the observer. The force is observed to be **less** by **O**, where the particle is moving relative to the observer. We will use this relation in chapter 22 on electromagnetism, together with length contraction equation [2]

Thus $F\,{}^{\Delta t}/_{\Delta t'} = F'$ and $F' = \dfrac{F}{[\,1 - u^2/c^2\,]^{1/2}}$ **Eqn. [6]**

21.13 The loss of simultaneity at a distance.

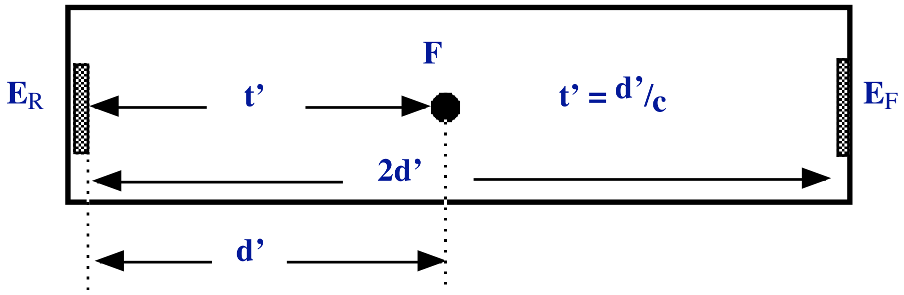
Simultaneous events occurring at the **same place** in space for one observer will be simultaneous for **all observers**. It can be shown that; if two events **separated in space** are simultaneous for one observer **O'**; they are not for a different observer **O** who is moving relative to **O'**.

In the situation below a spaceship is travelling at a speed **v** relative to **O**. Let us consider the events E_F and E_R , which are the arrival of two flashes of light at the front and rear of the ship. F is the event of the flash being emitted. We will use this example in the later section on spacetime intervals and four vectors.

Fig.21[xix] The frame of reference of O'.

O' does not detect any motion. He can be inside or outside the ship [it could be that the ship is stationary and **O** flies by].

O' at rest w.r.t. ship

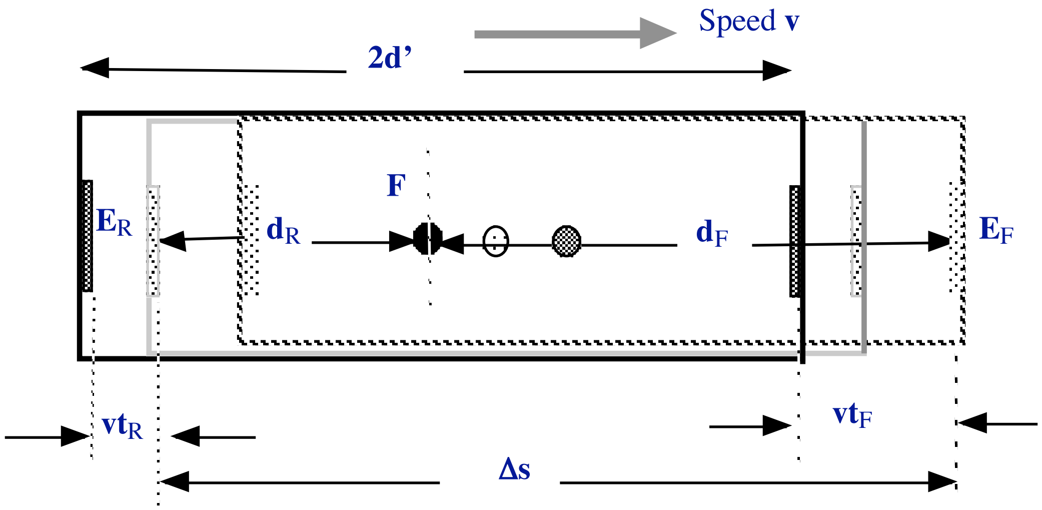


Our space traveller has put a light exactly in the middle of his ship and records the flash arriving at each end at the same time by his clocks. He concludes that the events E_F and E_R occurred simultaneously, but a distance **2d'** apart. The difference in the times of the two flashes arriving is zero **$\Delta t' = 0$.** The spatial separation of the two events E_F and E_R is **$\Delta s = 2d'$.** The events are separated in space **but not in time.**

A ground based observer **O** can also see the events E_F and E_R [flashes arriving at the detectors], but he will see the event E_R first. The distances that the light travels for **O** are **d_F** and **d_R** as shown in fig. 20[xviii] and the flash at the front will arrive after the flash hits the rear, since **d_F** is greater than **d_R .**

Fig.21[xx] The frame of reference of O.

Observer **O** moving left at **v.** Ship appears to move right at speed **v.**



Light from the flasher will have taken less time to reach the rear of the ship. Thus the two events were **not simultaneous** for **O'.**

$t_R = d - {}^{vt_R}/c$ thus **$t_R = {}^{d}/[c + v]$** similarly **$t_F = {}^{d}/[c - v]$**

Therefore the temporal separation of the events E_F and E_R is :-

$\Delta t = t_F - t_R = {}^{2vd}/[c^2 - v^2]$

The spatial separation of the two events is not **2d** because the ship moves in the time between the events :-

$\Delta s = d - vt_R + d + vt_F = 2d + {}^{2v^2d}/[c^2 - v^2]$

$\Delta s = {}^{2dc^2}/[c^2 - v^2]$

Hence the events are separated in space **and time.**

This leads to the concept of time as the **fourth dimension** and spacetime in the next chapter.